An initial corrector using H2 Approach for Youla Parametrisation via LMI Optimization

Ziani Salim
Electronics department, Mantouri University, Algeria
zianide_s@yahoo.fr
Djebabla Ali
Center University Laarbi Ben M'hidi Oum El Bouaghi, Algeria. tel. 213 (0)32476155
a_djebabla@yahoo.fr
Filali Salim
Electronics department, Mantouri University, Algeria
safilali@hotmail.com
Bououden Sofiane
Electronics department, Mantouri University, Algeria
usartsofiane@yahoo.fr

***Abstract-* This paper presents an approach by multiobjective optimization of the output feedback design in discrete time. The objective is to search a controller stabilizing the system with schedules charges temporal or frequential constraints. This is achieved by using the Youla parametrization based on initial corrector H2, combined with different Lyapunov functions; via LMI (Linear Matrix Inequality) optimization a comparison of approach is done with a initial corrector LQG, another goal of this work is also reducing the conservatism.**

***Index Terms-* Youla Parametrization , LMI Optimization, H2 control, Multiobjective control.**

I. Introduction

A LMI is a constraint of affining on the design variables, the characteristics of attenuation such as the placement of poles, robust stability, execution LQG, or of RMS, gains which can be expressed like LMIs. These characteristics define a multiobjective problem that can be solved numerically via convex optimization under LMI constraints[1]. The LMI optimization treats a problem with contradictory objectives, our objective is to found an optimal solution who is a compromise between all the defined objectives. The method of synthesis presented rests on the Youla parametrization [2,3]. Indeed, we use the fundamental properties of the Q-Parameter to present a methodology to obtain a representation of the inter-connected systems G, J and Q of closed loop $F_L(P,K)$ (linear Fractional Transformation or LFT Lower). We consider the H2 controller as an initial corrector for the Youla parametrization, Where the Q-Youla gives access to all the correctors K who stabilize the closed loop via the parameter Q. Hence it exist a corrector satisfying the schedule of conditions, we can find it by convex optimization[4]. This parameterization transform the initial problem into a convex LMI problem . This formalism is particularly adapted to the multi-criteria design because it possible to juxtapose the criteria without losing convexity [1,5,6]. In this work we can following this three steps to treat a problem of control by LMI convex optimization. The first one is the formulation of the initial problem to a optimization problem. The second is how to get a convex formulation, and the last one is the construct of the command law. Each stage of this process modifies the initial problem, and so induced a difference between the practical solution (found) and the theoretical optimal solution. Then the notion of the conservatism (Complexity / Calculability)[7] of the problem became another problem. The principle of multiobjective is to satisfy several criteria simultaneously.

II. Optimization problem

Is defined by:

$$\min_{K}\left(.\gamma_1,.\gamma_2 / \left[\|P*K\|_2<\gamma_1\right] \ et \ \left[\|P*K\|_\infty<\gamma_2\right]\right) \qquad (1)$$

3. Definition of some criteria

A. H∞ Norm [6,7]

Matrix characterization of the H∞ is represented by the inequality:

$$\exists X_1 = X_1^T > 0 \,/ \begin{bmatrix} -X_1^{T-1} & A & B & 0 \\ A^T & -X_1 & 0 & C^T \\ B^T & 0 & -\gamma_1 I & D^T \\ 0 & C^T & D & -\gamma_1 I \end{bmatrix} < 0 \qquad (2)$$

B. H2 Norm [1,4,7]

Is represented *by the inequality*: $\exists X_2 = X_2^T$ and $Y=Y^T > 0$ such as:

$$\begin{bmatrix} -X_2^{-1} & A & 0 \\ A^T & -X_2 & C^T \\ 0 & C & -I \end{bmatrix} < 0 \qquad \begin{bmatrix} -X_2^{-1} & B & 0 \\ B^T & -Y & D^T \\ 0 & D & -I \end{bmatrix} < 0 \qquad (3)$$

$$trace\,(Y) < \gamma_2^2$$

C. Property of α-stability [7]

A system (A,B, C, D) is α-stable if and only if:

$$\exists X_3 = X_3^T > 0 \,/ \begin{pmatrix} \alpha^2 X_3 & A^T \\ A & X_3^{-1} \end{pmatrix} > 0 \qquad (4)$$

III. Problem of LMI optimization [8,9]

$$\min_{\xi \in C} f(\xi)$$
$$C = \{\xi \in R^m \,/\, \forall\, x \in R^n, x^T F(\xi) x \geq 0\} \quad (5)$$
$$F(\xi) = F_0 + \sum_{i=1}^{m} \xi_i F_i$$

Some tools for LMI representation (formulation)

A. Lemma of Schur

The two inequalities are equivalents

$$\begin{cases} A(x)>0 \\ C(x)-B(x)^T A(x)^{-1} B(x)>0 \end{cases} \Rightarrow \begin{bmatrix} A(x) & B(x) \\ B(x)^T & C(x) \end{bmatrix} >0 \quad (6)$$

B. Modification by congruence

A and $\Pi \in C^{nxn}$ → if A>0 then $\Pi^T A \Pi \geq 0$ (7)

C. Lemma S-procedure

Is defined by this bijective relation:

$$\begin{cases} R^{nxn} \to R^{nxn} \\ \begin{bmatrix} W_i & Z_i \\ Z_i^T & Y_i \end{bmatrix} \mapsto \begin{bmatrix} R_i & S_i \\ S_i^T & T_i \end{bmatrix} = \begin{bmatrix} W_i^{-1} & -W_i^{-1} Z_i \\ -Z_i^T W_i^{-1} Y_i & -Z_i^T W_i^{-1} Z_i \end{bmatrix} \end{cases} \quad (8)$$

IV. Multicriteria problem

By using the following notations that refer to the figure1, the noted transfers of a T_i : $w_i \to z_i$ have as a representation of state [1]:

$$T_i = \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl} & D_{cl} \end{array}\right] = \left[\begin{array}{cc|c} A+B_u D_K C_y & B_u C_k & B_i + B_u D_K D_{yi} \\ B_K C_y & A_k & B_K D_{yi} \\ \hline C_i + D_{iu} D_K C_y & D_{iu} C_K & D_{ii} + D_{iu} D_K D_{yi} \end{array}\right] \quad (9)$$

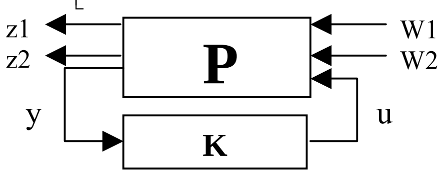

Fig.1. Closed loop system

Then closed loop system $T=(P*K)=LFT(P,K)=F_L(P,K)$ check the three following properties:

$$\|T_1\|_\infty < \gamma_1 \quad \|T_2\|_2 < \gamma_2 \quad LFT(P,K) \ \ is\, \alpha-stable \quad (10)$$

if and only if there are 4 matrices X_1, X_2, X_3 , Y such as : [1,6,7]

$$H\infty \quad \begin{bmatrix} -X_1 & X_1 A_{cl} & X_1 B_{1,cl} & 0 \\ A_{cl}^T X_1 & -X_1 & 0 & C_{1,cl}^T \\ B_{1,cl}^T X_1 & 0 & -\gamma I & D_{1,cl}^T \\ 0 & C_{1,cl} & D_{1,cl} & -\gamma I \end{bmatrix} < 0 \quad (11)$$

$$\begin{bmatrix} -X_2 & X_2 A_{cl} & 0 \\ A_{cl}^T X_2 & -X_2 & C_{2,cl}^T \\ 0 & C_{2,cl} & -I \end{bmatrix} < 0$$

$$H2 \quad \begin{bmatrix} -X_2 & X_2 B_{2,cl} & 0 \\ B_{2,cl}^T X_2 & -Y & D_{2,cl}^T \\ 0 & D_{2,cl} & -I \end{bmatrix} < 0 \quad (12)$$

$$trace\,(Y) < \gamma_2^2$$

$$\alpha\text{-stability} \quad \begin{pmatrix} \alpha^2 X_3 & A_{cl}^T X_3 \\ X_3 A_{cl} & X_3 \end{pmatrix} > 0 \quad (13)$$

The inequalities of the multiobjective problem (11-13) are not linear on the unknown variables, and there does not exist today of methods to solve this kind of problem. It is thus necessary to transform it, while preserving its characteristics. We keep the Lyapunov function considered different for each criteria (Xi, i=1,2,3 for H2, H∞, α-stability).

V. Optimization of the parameter of Youla

The standard representation is represented in figure2 [2, 3]

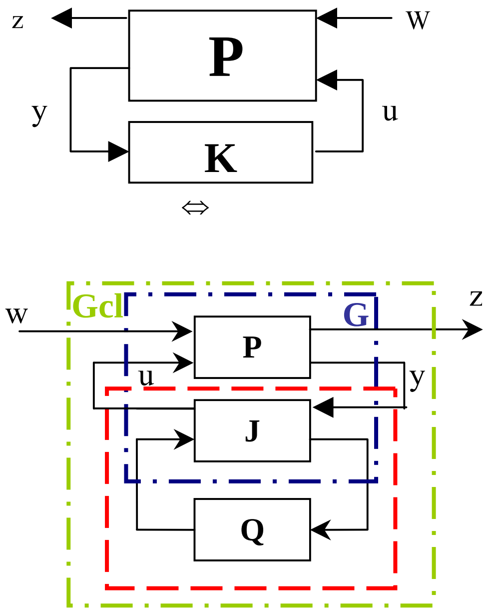

Fig.2. General form of the parameterization of Youla

$$P = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (14)$$

$$K = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \quad (15)$$

Defined the two dual algebraic Riccati equations[10,11]

$$A^T X_2 + X_2 A - X_2 B_2 B_2^T X_2 + C_1^T C_1 = 0 \quad (16)$$

$$A Y_2 + Y_2 A^T - Y_2 C_2^T C_2 Y_2 + B_1 B_1^T = 0 \quad (17)$$

The two gains F_2 and L_2 such as (A, B_2) is stabilizable and $A,C_2)$ is detectable, or there the two evolution matrices $(A-B_2F_2)$ and $(A-L_2C_2)$ (18) are stable, we can computed by:

$$F_2 = -B_2^T X_2 \qquad (19)$$

and

$$L_2 = -Y_2 C_2^T \qquad (20)$$

The H2 corrector is defined by :

$$J_2 = \left[\begin{array}{c:cc} A_2 + B_2 F_2 + L_2 C_2 & -L_2 & B_2 \\ \hdashline F_2 & 0 & I \\ -C_2 & I & 0 \end{array}\right] \qquad (21)$$

According to the fundamental property of the Youla parametrization $G_{22}=0$ which expresses that all the transfer of the closed loop system are linear on Q [2,3] :

$$T = P*K = P*K = G*Q \qquad (22)$$
$$= P*J_2*Q = G_{11} + G_{12}Q(I - G_{22}Q)^{-1}G_{21} = G_{11} + G_{12}*Q*G_{21}$$

It possible to obtain a representation of state which is simultaneously in form commendable and observable in the following form [2, 3]:

$$G = P*J_2$$

$$G = \left[\begin{array}{cc:cc:c} A_1 & A_3 & B_{11} & B_{21} & B_{\hat{u}} \\ 0 & A_2 & B_{12} & B_{22} & 0 \\ \hdashline C_{11} & C_{12} & D_{11} & D_{12} & D_{1\hat{u}} \\ C_{21} & C_{22} & D_{21} & D_{22} & D_{2\hat{u}} \\ \hdashline 0 & C_{\hat{y}} & D_{\hat{y}1} & D_{\hat{y}2} & 0 \end{array}\right] \qquad (23)$$

To obtain the Youla parameterization returns to calculate: The system of interconnection J and Q such as J*Q=K figure 2 stabilizing for P, for our approach is to fix J and find Q optimal via LMI optimization.

VI. Linearization of the matrix inequalities

The system G of the figure2 is asymptotically stable, and Q a static output-feedback. We will show that there is a LMI formulation with the control problem H2, H∞ and α-stability. This result is obtained by matrix handling. Let us consider the matrix characterizations of the H2 and H∞ norms of the closed loop system T (24); taking into account property 3 of the Youla parametrization, the representation of state of the system T has the following form: [3]

$$T = \begin{bmatrix} A_{Gcl} & B_{Gcl} \\ C_{Gcl} & D_{Gcl} \end{bmatrix} = \left[\begin{array}{cc:c:c} A_1 & A_3 + B_{\hat{u}} Q C_{\hat{y}} & B_{1,1} + B_{\hat{u}} Q D_{\hat{y}1} & B_{2,1} + B_{\hat{u}} Q D_{\hat{y}2} \\ 0 & A_2 & B_{1,2} & B_{2,2} \\ \hdashline C_{1,1} & C_{1,1} + D_{1\hat{u}} Q C_{\hat{y}} & D_{11} + D_{1\hat{u}} Q D_{\hat{y}1} & D_{12} + D_{1\hat{u}} Q D_{\hat{y}2} \\ C_{2,1} & C_{2,2} + D_{2\hat{u}} Q C_{\hat{y}} & D_{21} + D_{2\hat{u}} Q D_{\hat{y}1} & D_{22} + D_{2\hat{u}} Q D_{\hat{y}2} \end{array}\right] \qquad (24)$$

Characterizations by matrix inequalities of the H∞ and H2 norms exposed to § 4 are applied to T and they are not linear on the decisions variables (Xi Lyapunov functions and the gain Q). For example the term $X_1.A_{Gcl}$ (11) fact of intervening a term where intervenes in the same product X_1 and Q. It is thus necessary to modify the problem to transform it into a convex problem of optimization. So the Xi is the Lyapunov function associate on the criteria i (i=1,2,3 respectively H∞, H2, α-stability) it partitioned in the same proportions that the evolution matrix A_G:

$$X_i = \begin{bmatrix} W_i & Z_i \\ Z_i^T & Y_i \end{bmatrix} \quad according \quad to \quad A_{\tilde{G}} = \begin{bmatrix} A_1 & A_3 \\ 0 & A_2 \end{bmatrix} \qquad (25)$$

Using the S-procedure lemma (bijective change of variable), and the propriety of congruence lemma with taking the following matrix:

$$M_i = \begin{bmatrix} R_i & 0 \\ S_i^T & I \end{bmatrix} \qquad (26)$$

By applying these successions of stages to the system T. The matrices characterizations of the three criteria described to the §4, we obtain the following LMI:

A. Problem of H∞

By using the congruence: $\Pi_1 = diag(M_1\ M_1\ I\ I)$ (27)

We obtain from(11):

$$\begin{bmatrix} -R_1 & 0 & A_1R_1 & A_1S_1 - S_1A_2 + A_3 + B_{u\wedge}QC_{y\wedge} & B_{1,1} + B_{u\wedge} + B_{u\wedge}QD_{y\wedge 1} - S_1B_{1,2} & 0 \\ 0 & -T_1 & 0 & T_1A_2 & T_1B_{1,2} & 0 \\ * & * & -R_1 & 0 & 0 & R_1^TC_{1,1}^T \\ * & * & 0 & -T_1 & 0 & C_{1,2}^T + C_{y\wedge}^TQ^TD_{1u\wedge}^T + S_1^TC_{1,1}^T \\ * & * & * & * & -\gamma_1 I & D_{11}^T + D_{y\wedge 1}Q^TD_{1u\wedge}^T \\ * & * & * & * & * & -\gamma_1 I \end{bmatrix} < 0 \qquad (28)$$

B. Problem of H2

By using the congruence lemma by the two matrix: Π_{21}, Π_{22}

$\Pi_{21} = diag(\ M_2\ \ M_2\ \ I\ )$ (29)

and $\Pi_{22} = diag(\ M_2\ \ I\ \ \ I\ )$ (30)

We obtain from (12) :

$$\begin{bmatrix} -R_2 & 0 & A_1R_2 & A_1S_2 - S_2A_2 + A_3 + B_{u\wedge}QC_{y\wedge} & B_{1,1} + B_{u\wedge} + B_{u\wedge}QD_{y\wedge 1} - S_2B_{1,2} & 0 \\ 0 & -T_2 & 0 & T_2A_2 & T_2B_{1,2} & 0 \\ * & * & -R_2 & 0 & 0 & R_2^TC_{2,1}^T \\ * & * & 0 & -T_2 & 0 & C_{2,2}^T + C_{y\wedge}^TQ^TD_{2u\wedge}^T + S_1^TC_{2,1}^T \\ * & * & * & * & * & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} -R_2 & 0 & B_{2,1} + B_{u\wedge}QD_{y\wedge 2} - S_2B_{2,2} & 0 \\ 0 & -T_2 & 0 & 0 \\ * & * & -Y & D_{22}^T + D_{y\wedge 2}^TQ^TD_{2u\wedge}^T \\ * & * & 0 & -I \end{bmatrix} < 0$$

$$trace(Y) < \gamma_2^2 \qquad (31)$$

C. Problem of α-stability

By using congruence by the matrix :

$\Pi_3 = diag(\ I\ \ M_3\ )$ (32)

we obtain from (13)

$$\begin{bmatrix} \alpha^2 R_3 & * & * & * \\ 0 & \alpha^2 T_3 & * & * \\ A_1 R_3 & A_1 S_3 - S_3 A_2 + B_2 Q C_2 + A_3 & R_3 & * \\ 0 & T_3 A_2 & 0 & T_3 \end{bmatrix} > 0 \qquad (33)$$

These various inequalities (28), (31), (33) are indeed linear on the variables of decisions R_1, S_1, T_1, Q and γ_1 for the problem H∞ and R_2, S_2, T_2, Q and γ_2^2 for the problem H_2 and R_3, S_3, T_3, Q for the problem α-stability. The three problems H2 and H∞ and α-stability are coupled by the static output feedback Q and the different Lyapunov functions.

Note:

The value of α-stable was selected to force the dominant poles of the closed loop.

The sensitivity is defined by $S=(I+KG)^{-1}$ [3]. (34)

VII. Application

We consider the system P defined by : [12]

$$P(z)=\frac{0.2879z^2+0.03516z-0.2217}{z^3-2.158z^2+1.874z-0.6908} \qquad (35)$$

A. Schedule of conditions

1. Response time tr<1.8s, and time of rejection of disturbance t_{rd}< 1.8 s
2. Max of sensitivity function disturbance -output |S|<15db
3. Max of sensitivity function disturbance -command |KS|<12db

B. Implementation by the introduction of the weight functions

to achieve the desired objectives. For that, one considers figure 4 in which the error e is weighted by the filter W3(p), the control u by W2(p), and the entry of disturbance b is weighted by the filter W1(p), one can puts the whole in the following form:

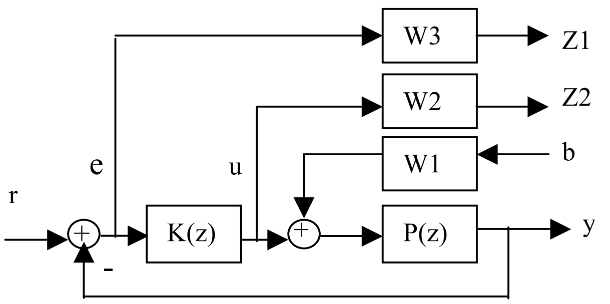

Fig.4. Block diagram of the augmented system

While taking **:**

$$w1=0.82\frac{1+0.08z}{1+0.09z}$$

$$w2=90\frac{1+0.086z}{1+0.096z}$$

$$w3=675e-5\frac{1e-8-370z}{1+0.09z}$$

Note.1.

The frequential response for each functions S and KS is a constraint, which depends on the filter selected:

$$\|W1S\|_\infty<\gamma \iff \forall\omega\in R \quad |S(j\omega)|<\frac{\gamma}{|W1(j\omega)|} \; ;$$

$$\|W2KS\|_\infty<\gamma \iff \forall\omega\in R \quad |K(j\omega)S(j\omega)|<\frac{\gamma}{|W2(j\omega)|} \qquad (36)$$

Note.2.

To reduce the conservatism from the complexity (and calculability) point of view of the problem one satisfies to optimize γ such as $\gamma=\gamma_1+\lambda\gamma_2^2$ / $\lambda=0.01\in[0, 1]$ without losing the convexity of LMI problem. The following results are obtained:

Table1.

Recapitulative

Recapitulative	With H2 initial corrector	With LQG initial corrector
Nbr of matrix inequalities	6	6
Nbr of decision variables	195	195
Nbr of objectives	1	1
Lyapunov F[cts]	Different	Different
Structure of Lyapunov F[cts]	Toeplitz	Toeplitz
α-stable value	0.88	0.88
Response time	Tr=1.5sec	t_r= 1.8 sec
Rejection of disturbance time	$T_{rej/dist}$=1.5 sec	$T_{rej/dist}$=1.5 sec
Overschoot	D=1.10 db	D=1.20 db
Time computing LMI	47.5101 sec	8.8001 sec
Max of the sensitivity S	|S|=14.8 db	|S|=7.82 db
Q_{opt} γ_{opt}	Q_{opt}= 4.0082 γ_{opt}= 2.9162	Q_{opt}= 0.9551 γ_{opt}= 1.6327

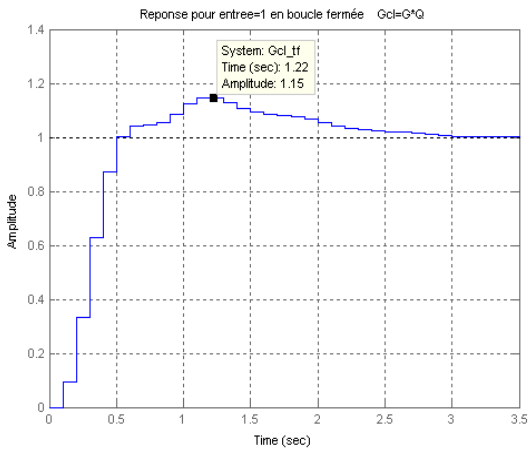

Fig5. Response of the system with LQG corrector initial

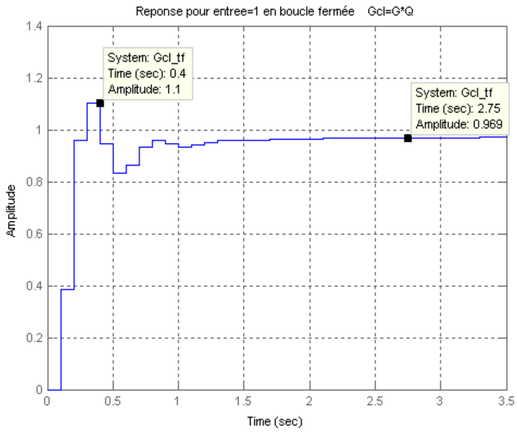

Fig.6. Response of the system with H2 corrector initial

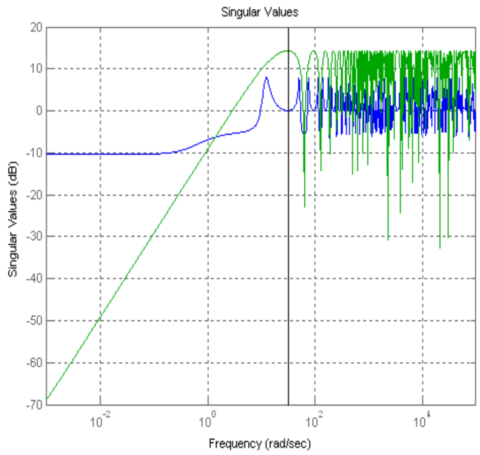

Fig7. Value singular of sensitivity S (Blue) and the function 1/W3 (green) With LQG corrector initial

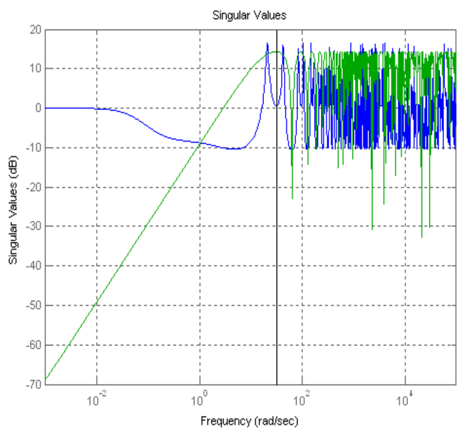

Fig8. Value singular of sensitivity S (Blue) and the function 1/W3 (green) With H2 corrector initial

VIII. CONCLUSION

From the properties of the Youla parametrization we can to present a methodology who it possible to obtain a representation of the inter-connected systems of the closed loop. This parametrization can to ensure also the convexity of the problem (optimal solution), under an initial H2 corrector which gave us a good results. The solution given by the LMI optimization for the defined objectives is a compromise between all objectives, and this design in term of optimization is similar to the approach of the optimality defined by Pareto. Moreover the unacceptable computing time and the occupied memory capacity increase the problem of conservatism. The quality of the results (responses) depends on the initial corrector selected. In this work the optimal response of the closed loop system was enhancing by ours initial corrector.

REFERENCES

[1]. A. Molina-Cristobal, I.A. Grifin. P.J. Fleming and D.H. Owens, Linear matrix inequalities and evolutionary optimization in multiobjective control, IJSS, V.37, N°8.20, June 2006

[2]. P. Rodríguez, D. Dumur, " Robustification of GPC controlled system by convex optimization of the Youla parameter ", Proceedings IEEE Conference on Control Applications, Glasgow, Sept. 2002.

[3]. B. Clément, " Utilisation de la Paramétrisation de Youla pour la commande ", Service Automatique de Supélec, Goupe Commande Robuste, mars (2000).

[4]. Genci Capi, Masao Yokota, Optimal multi-criteria Humanoid robot gait synthesis an evolutionary approach, IJCIC, V.2, N°6, Dec. 2006.

[5]. Chung Seop Jeong, Edwin E. yaz, nonlinear observer design with general criteria, IJCIC, V.2, N°4, August 2006.

[6]. Fuzhong Wang, Qingling Zhang, LMI-Based reliable H∞ filtering with sensor failure, IJICIC, V2, N°4, August 2006.

[7]. B. Clement, G. Duc; Multiobjctive Control via youla parametrisation and LMI optimisation : Application to fexible arm, IFAC Symposium on Robust Control and Design, Prague, juin (2000

[8]. Pascal Gahinet , Arkadi Nemirovski, Alan J. Laub, Mahmoud Chilali; LMI Control Toolbox Ver. 3. 2001

[9]. Boyd, S.P., El Ghaoui, L., Feron, E., and Balakrishnan, V., Linear Matrix Inequalities in Systems and Control Theory, Philadelphia, PA, SIAM, Vol. 15, SIAM Studies in Applied Mathematics, 1994.Vidyasagar, M., Control system synthesis :A factorization Approach. The MIT press, Cambridge etc., 1985

[10]. Peres, P., Geromel, J., and Souza, S.,, "Optimal H2 control by output feedback", Proc. IEEE Conf. On Decision and control, 1993.

[11]. F. Leibfritz, "An LMI-based algorithm for designing suboptimal static H2/H∞ output feedback controllers", SIAM J. Control Optim. Vol 39, 2001.

[12]. Richard C.Dorf, R. H. Bishop, Modern control systems,7th edition, Addison-Wesley publishing company, England, PP 434, 1995.