Optimisation of Fuzzy Predictive Functional Control using coordination techniques

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Abstract— **Predictive functional control (PFC), belongs to the family of predictive control techniques. It has been demonstrated as a powerful algorithm for controlling process plant. In this paper, PFC strategy is extended to nonlinear processes. The predictive functional control is combined with a fuzzy model of the process and formulated in the state space domain. The process model is decomposed into subsystems each described by a fuzzy rules. In controller design, prediction errors and control energy is minimised trough a two layered iterative optimization process. The lower layer finds local control policies for each sub system. The objective of the upper layer is to find a near optimum for the overall system trough coordinating the subsystems. The performance of the FPFC approach is demonstrated trough an example of CSTR process.**

***Index Terms*—Fuzzy identification, predictive control, decomposition-coordination, nonlinear systems.**

I. INTRODUCTION

Predictive functional control frequently abbreviated to PFC, is a form of predictive control which belongs to the classical family of model based predictive control since it fulfills the following basic generic principles [1]:

-Internal model: used for prediction
-Reference trajectory: to specify the future closed loop behaviors
-Structure future manipulated variable and algorithmic solver
-Modeling error compensation to take into account prediction error.

PFC can use many forms of internal model, including state space, input output, finite input response (FIR), fuzzy rule…However the main distinguishing feature of PFC over other MPC algorithms is that the internal models used are independent internal models, which depend solely on the process input. Perhaps the second distinctive feature of PFC is that the future input is assumed to be a linear combination of a few simple basis functions. In principle these could be any appropriate functions.

There have been many propositions to generalize linear predictive control algorithms to nonlinear systems through the use of Takagi Sugeno fuzzy inference system [2]. Two approaches can be identified. In the first, the TS fuzzy inference systems is used to combine local linear models, producing an overall linear time variant predictive model which is used to derive the control law [3,4] . In the second, local linear model based predictive controllers may be computed for each rule and the local controllers are merged to produce the overall controller. In this case, the method used for merging the local controllers is usually based on the weight associated with each local model [5,6,7]. However this does not guarantees optimality of the control law. To alleviate this problem, it has been suggested in [8] to construct a hierarchical control algorithm that guarantees optimality for fuzzy predictive control. In this work, we apply this hierarchical approach to design predictive functional control law for non linear system based on Takagi Sugeno fuzzy model. This paper is organized as follows: in the second section, we introduce the fuzzy identification, in the third section we develop the hierarchical approach and in the fourth section we present the simulation results.

II. FUZZY IDENTIFICATION

Consider a single-input single output (SISO) non linear system S. The model of the system is decomposed into *P* submodels such that each demonstrates a linear behavior. By Takagi-Sugeno's modeling methodology [2], it can be written as:

$$R^j : if\ x_1 is\ A_1^j\ and..and\ x_N is\ A_N^j\ then \quad y = f^j(x_1,....,x_N) \tag{1}$$

Where $x_1,...,x_N$ are the inputs, A_i^j is a subset of the input space, y is the output, and f^j is a linear function of the inputs.

In our case the local fuzzy linear sub model corresponding to fuzzy rule $\boldsymbol{R^i}$, is established in a discrete time representation. It is written as:

R^i : if *u(k)* is A^i then

$$y^i(k+1) = a_1^i(k)y(k) + a_2^i(k)y(k-1) + a_3^i(k)y(k-2) + b^i(k)u(k-D) + r^i, \quad (2)$$

Where $y^i(k+1)$ is the output of the *ith* local linear submodel corresponding to ith fuzzy rule and $y(k), y(k-1), y(k-2), u(k-D)$ are the inputs to the fuzzy model. ***D*** stands for the dead time expressed by the number of samples, A^i are antecedent fuzzy sets and *u(k)* is the antecedent variable. The output of the overall dynamical model is obtained by merging all local linear submodel using fuzzy mean defuzzification method:

$$y_p(k+1) = \sum_{i=1}^{P} \beta_i(k)(a_{1i}y(k) + a_{2i}y(k-1) + a_{3i}y(k-2) + b_i u(k-D) + ri), \quad (3)$$

Where *P* stands for the number of rules and $\beta_i(k)$ is the normalized degree of fulfillment of *i th* rule at *k th* step:

$$\beta_i(k) = \frac{\mu_{A_i}(y(k))}{\sum_{i=1}^{P} \mu_{A_i}(y(k))} \quad (4)$$

With:

$$\sum_i \beta_i(k) = 1$$

The parameters of the fuzzy model are obtained on measured input output data using classical least square optimization method.

$$\theta_i = (\Psi_i^T \Psi_i)^{-1} \Psi_i^T Y^i \quad (5)$$

Where:

$$\theta_i^T = [a_{1i} \quad a_{2i} \quad a_{3i} \quad b_i \quad r_i] \quad (6)$$

ψ_i and Y^i are matrices of data.

Vectors θ_i can be grouped in a matrix of parameters

$$\Theta = [\theta_1 \quad \theta_2 \quad . \quad . \quad \theta_P], \quad (7)$$

Where i th column represents the parameter vector the fuzzy model can be written in the following form of the i th subsystem:

$$y^i(k+1) = \tilde{a}_1^i(k)y(k) + \tilde{a}_2^i(k)y(k-1) + \tilde{a}_3^i(k)y(k-2) + \tilde{b}^i(k)u(k-D) + \tilde{r}^i, \quad (8)$$

i=1…P

III. PREDICTIVE FUNCTIONAL CONTROL BASED ON FUZZY MODEL

The design goal of predictive control is to predict the future behavior of the process over certain horizon using the dynamic model and obtaining the control actions to minimize a certain criterion, generally

$$J(u,k) = \sum_{N1}^{N2} (y_m(k+j) - y_r(k+j))^2 + \lambda \sum_{j=1}^{Nu} u^2(k+j) \quad (9)$$

$y_m(k+j), y_r(k+j), u(k+j)$ Are respectively *j*-step ahead predictions of the process output, the reference trajectory and the control signal, respectively. Parameter λ is the weight of the control signal energy.N_1,N_2 and N_u are minimum and maximum output predition horizon and control horizon respectively.

PFC is designed in the time domain. Altough in principle it is similar to the classical model based predictive control its formulation is different. It is based on the idea of coincidence points in a prediction horizon *H* and the minimization of cost function (9) is not explicit [1].

In the following, we develop the hierarchical control design for fuzzy PFC. The whole design is decomposed into the derivation of P local functional predictive controllers which will be coordinated to derive a global optimal control strategy. The local controllers are based on linear submodels of the form (2) and are produced by the lower layer.

A) Lower layer design:

At this layer we consider all P linear submodels, then for the *j th* submodel we find the associated control signal. We first rewrite (2) in state space form.define the optimization problem as follows:

$$X^i{}_m(k+1) = \tilde{A}^i{}_m X^i{}_m(k) + \tilde{B}^i{}_m u^i(k) + R^i{}_m + \varepsilon^i(k) \quad (10)$$

The state vector $x_m(k)$ is

$$x^i{}_m(k) = \begin{bmatrix} y^i{}_m(k) \\ y^i{}_m(k-1) \\ y^i{}_m(k-2) \end{bmatrix} \quad (11)$$

The matrix $\tilde{A}_m, \tilde{B}_m, \tilde{R}_m$ and $\tilde{C}_m$ become:

$$\tilde{A}^i{}_m = \begin{bmatrix} \tilde{a}^i{}_1 & \tilde{a}^i{}_2 & \tilde{a}^i{}_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (12)$$

$$\tilde{B}^i{}_m = \begin{bmatrix} \tilde{b}^i \\ 0 \\ 0 \end{bmatrix} \quad (13)$$

$$\tilde{R}^i{}_m = \begin{bmatrix} (\tilde{r}^i + \varepsilon^i(k)) \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

$$\tilde{C}^i{}_m = [1 \quad 0 \quad 0] \quad (15)$$

Where $\varepsilon^i(k)$ serve for system coordination and is determined at the upper layer.

In order to derive the local predictive functional control law, we consider the prediction model of the *i th* subsystem. The *H* step-ahead prediction can be written

$$y^j_m(k+H)=\tilde{C}m^j(\tilde{A}^{iH}_m X_m(k)+ (\tilde{A}^{iH-1}_m+...+\tilde{A}^i_m+I)\times(\tilde{B}_m u(k)+\tilde{R}^i_m)) \quad (16)$$

With $u(k)=u(k+1)=...=u(k+H-1)$

The sum of the powered $\tilde{A}_m$ matrices can be simplified as

$$\tilde{A}^{iH-1}_m+....+\tilde{A}^i_m+I=(\tilde{A}^{iH}_m-I)(\tilde{A}^i_m-I)^{-1} \quad (17)$$

The closed loop response should be similar to the reference trajectory, which is the output of the reference model:[6]

$$X_r(k+1)=A_r X_r(k)+B_r w(k)$$
$$y_r(k)=C_r X_r(k) \quad (18)$$

Matrices Ar, Br and Cr have to be chosen to fulfill the following equation:

$$C_r(I-A_r)^{-1}B_r=1 \quad (19)$$

In the same manner, the H step-ahead prediction of the reference model can be written as:

$$y_r(k+H)=C_r(A_r^H x_r(k)+(A_r^H-I)(A_r-I)^{-1}B_r w(k)) \quad (20)$$

The main idea of FPFC is the equivalence of the objective increment of the process and the model output increment. The objective increment Δ_P is defined as the difference between predicted reference trajectory $y_r(k+H)$ and actual output signal:

$$\Delta_p=y_r(k+H)-y^i{}_p(k) \quad (21)$$

$$\Delta_p=C_r(A_r^H x_r(k)+(A_r^H-I)(A_r-I)^{-1}B_r w(k))-y_p(k) \quad (22)$$

The model output increment Δ_m is defined by:

$$\Delta_m=y^i{}_m(k+H)-y^i{}_m(k)$$
$$\Delta_m=C_m(\tilde{A}^{iH}_m x_m(k)+(\tilde{A}^{iH}_m-I)(\tilde{A}^i_m-I)^{-1}(\tilde{B}_m u(k)+\tilde{R}_m))-y^j_m(k) \quad (23)$$

From the above equations and the goal of FPFC, this is described with the following:

$$\Delta_m=\Delta_p \quad (24)$$

By deriving the control variable *u (k)*, the control law given by:

$$u^i(k)=\frac{C_r A_r^H x_r(k)+C_r(A_r^H-I)(A_r-I)^{-1}B_r w(k)-y^i{}_p(k)}{\tilde{C}^i{}_m(\tilde{A}^{iH}_m-I)(\tilde{A}^i_m-I)^{-1}\tilde{B}^i{}_m}- \frac{\tilde{C}^i{}_m\tilde{A}^{iH}_m x^i{}_m(k)-\tilde{C}^i{}_m(\tilde{A}^{iH}_m-I)^{-1}\tilde{R}^i{}_m-y^i{}_m(k)}{\tilde{C}^i{}_m(\tilde{A}^{iH}_m-I)(\tilde{A}^i{}_m-I)^{-1}\tilde{B}^i{}_m} \quad (25)$$

The control signal $u^i(k)$ is produced by the ith linear submodel. The following set includes the information which will be transmitted to the upper layer.

$$S^i_{L\to U}=\{y^i(k+H),u^i(k)\} \quad (26)$$

B) Upper layer design:

The upper layer coordination targets the identification of globally optimal control policies through coordinating $\varepsilon^i(k)$ for each of the P local submodels: $\varepsilon^i(k)$ represent the difference between *y(k+H)* and y^i *(k+H).*

From the lower layer, the local information of output and control in $S^j_{L\to U}$ is transmitted to the upper layer.

At the upper the error variables are evaluated as $e^j(k+H)-y^j(k+H)$

These values are compared with those for the same error variable calculated in the last iteration, say $\varepsilon^j(k+H)$ and compared with the smallest tolerable error ζ. Then if

$$\sum_{j=1}^{P}\left|e^j(k+H)-\varepsilon^j(k+H)>\zeta\right| \quad (27)$$

the control policies are not optimal and need be modified at the local layer this can be accomplished in a new iterative process by sending down the set $S^j_{U\to L}$ for each subsystem. This exchange of information between lower and upper layer is repeated during the sampling period until :

$$\sum_{j=1}^{P}\left|e^j(k+H)-\varepsilon^j(k+H)<\zeta\right| \quad (28)$$

in which case the global control policy derived from the optimal local control policies at the lower layer is considered as near optimal. The control signal is then obtained by merging the local control policies according to fuzzy mean defuzzification method, that is:

$$\sum_{i=1}^{P}\beta_i u^i(k)$$

with β_i given by (4) by (4) and satisfying $\sum_i \beta_i(k)=1$

IV. RESULTS OF SIMULATION

In this section we consider the application of the algorithm to the control of the concentration on a continuous stirred tank reactor. The CSTR is a process highly nonlinear, which very common in chemical and petrochemical plants. In the process, an irreversible, exothermic reaction A $\rightarrow$ B takes place in a constant volume reactor cooled by single coolant flow. The process is modelled by the following equations:

$$\frac{dC_A(t+d)}{dt}=\frac{q(t)}{V}(C_{A0}(t)-C_A(t+d)-k_0C_A(t+d) \times\exp\left(\frac{-E}{RT(t)}\right) \quad (29)$$

$$\frac{dT(t)}{dt}=\frac{q(t)}{V}(T_0(t)-T(t))-\frac{(-\Delta H)k_0C_A(t+d)}{\rho C_p} \times\exp\left(\frac{-E}{RT(t)}\right)+\frac{\rho_c C_{pc}}{\rho C_p V}q_c(t)\left\{1-\exp\left(\frac{-hA}{q_c(t)\rho C_p}\right)\right\} \times(T_{c0}(t)-T(t)). \quad (30)$$

The objective of the design is to control the concentration of A, C_A *(t)* by manipulating the coolant flow rate q_c *(t)* the nominal parameter values of the process are given in Table1.

For fuzzy modelling, the above nonlinear model is used to produce input-output time series data. The sampling time is set to 0.083min (5s). The data is then used to develop a global fuzzy model as follows:

R^1: IF q_c is Q^1
THEN $C^1_A(n+1)=a_{11}C_A(n-1)+ a_{12} C_A(n-2)+ a_{13} C_A(n-3)+b_1q_c(n)+r_1$

R^2: IF q_c is Q^2
THEN $C^2_A(n+1)=a_{21}C_A(n-1)+ a_{22} C_A(n-2)+ a_{13} C_A(n-3)+b_2q_c(n)+r_2$

The fuzzy model is structurally very simple and requires only two fuzzy rules, fig.2 shows the fuzzy sets Q^1 and Q^2.
Fig. 3 shows the open loop response with various step changes in the coolant flow rate: the identification of the fuzzy model can nearly perfectly describe the process dynamic behaviour .
The vector of parameters of i^{th} rule is obtained by using the least squares method:

a_{11} = -0.9090 a_{21} =-0.9264
a_{12} =-0.1444 a_{22} =-0.1383
a_{13} = 0.0615 a_{23} = 0.0680
b_1 = 0.0002 b_2 =0.0008
r_1 = -0.0013 r_2 =-0.0005

TABLE I
SPECIFICATION OF THE CSTR

Process variable	Normal operation condition
Measuredproduct concentration(C_A)	0.1mol/l
Reactor temperature(T)	438.54K
Coolant flow rate(qc)	103.41l/min
Process flow rate (q)	100.0 l/min
Feed concentration(C_{A0})	1 mol/l
Feed temperature(T0)	350.0K
Inlet coolant temperature (Tc0)	350.0K
CSTR volume(V)	100l
Heat transfer term(hA)	7.2×10^5 cal/(min.K)
Reaction rate constant(k0)	7.2×10^{10}min^{-1}
Activation energy term(E/R)	1×10^4 K
Heat of reaction (-ΔH)	-2×10^5 cal/mol
Liquid density (ρ, ρ_c)	1×10^3g/l
Specific heats(Cp,Cpc)	1 cal/(g.K)

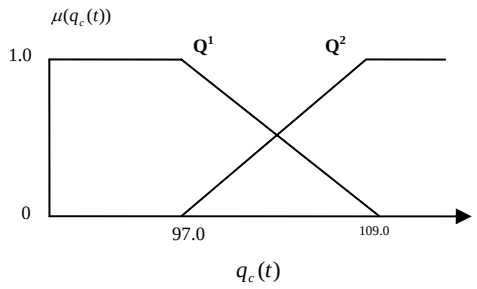

Fig 1: Fuzzy sets Q^1 and Q^2 for FIs R^1 and R^2, respectively

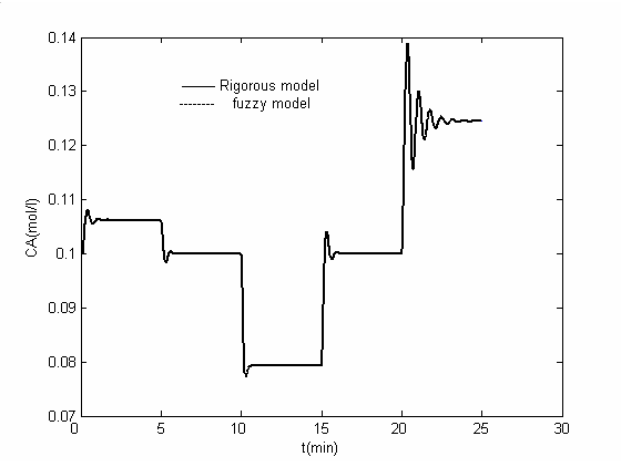

Fig..2 open loop composition response of the CSTR process

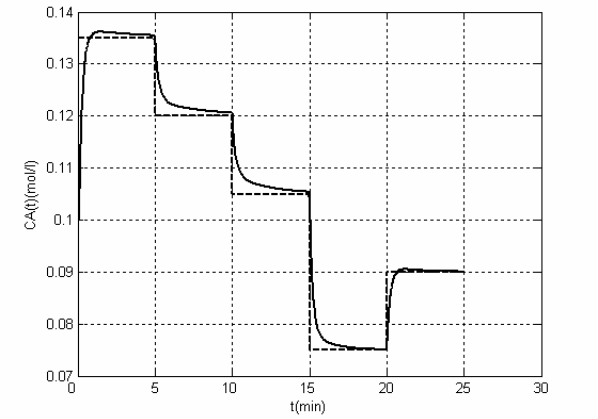

Fig.3 Concentration set point tracking of the CSTR process

We study the control quality the FMPC controller by a series of simulations. In testing the set point tracking capability, the set point of C_A was changed from the nominal operating point 0.1 mol/l to 0.135, to 0.12, to 0.105, to 0.75, and then to 0.09 (Fig 3 , dash line) . The dynamic response of the system is depicted in the same figure.
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V. CONCLUSION

.A fuzzy functional predictive controller for nonlinear systems based on hierarchical design has been developed. At the lower level, local controller are designed for each rule of a Fuzzy Takagi Sugeno model. At the upper level coordination is performed using error variables. The approach reduces the computation time of nonlinear model predictive control and allows online applications. The method is applied for the control of concentration of a CSTR with encouraging results.

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